Theoretical Error Analysis of the Accuracy of Focal Depth Determination of Near Earthquakes

Zhang Chaojun 1,2, Zhang Xiaodong 1, Miao Chunlan 1, Ding Qiuqin 1, Zhang Aiwu 1 and Hu Bin 1

1) China Earthquake Networks Center  2) Institute of Earthquake Science

Focal depth is one of the most difficult seismic parameters to determine accurately in seismology. The focal depths estimated by various methods are uncertain to a considerable degree which affects the understanding of the source process. The influence of various factors on focal depth is non-linear. The influence of epicentral distance, arrival time residual and velocity model (crust model) on focal depth is analyzed based on travel time formula of near earthquakes in this paper. When wave propagation velocity is constant, the error of focal depth increases with the increase of epicentral distance or the distance to station and the travel time residual. When the travel time residual is constant, the error of focal depth increases with the increase of the epicentral distance and the velocity of seismic wave. The study also shows that the location error perhaps becomes bigger for shallower earthquakes when the velocity is known and the travel time residual is constant. The horizontal error caused by location accuracy increases with the increase of the epicentral distance, the travel time residual and the velocity of seismic waves, thus the error of focal depth will increase with these factors. On the other hand, the errors of focal depth will lead to change of the origin time, therefore resultant outcomes will all change.

Key words: Focal depth $h$; Location accuracy; Error

INTRODUCTION

Focal depth is one of the most basic parameters which describes the seismic source and gives the specific location of the earthquake occurring in the Earth’s interior. It is of important significance for the understanding of physical and chemical conditions of earthquake preparation and occurrence, and the building-up and releasing of seismic energy as well as the study of active
tectonic settings. Seismologists use it to estimate the thickness of the lithosphere plate and to gain deeper insight into the detailed tectonic processes and to explore the mechanical mechanisms and process of earthquake occurrences. The accurate determination of the focal depth relates to a series of important issues such as correctly understanding the source process, fault structures, crust and mantle structures, stress field, plate movement and so on (Gao Yuan, 1997). In fact, for the study of any seismic event, the study of macro effects of an earthquake to the identification of earthquakes and nuclear explosions, it is a must to know the focal depth.

The precision of focal depth is still a thorny problem in modern earthquake catalogs. It has almost become one of the least accurate parameters (Gao Yuan, 1997). This is because earthquake location is influenced by the double impact resulting from observation errors of seismic phase identification and the error between theoretical crustal models and real earth crust, which in practical work is hard to identify from each other (Billings et al., 1994).

Many scholars use different methods to compute the focal depth such as: (1) Using the characteristics that the slowness changes of travel time curve are very sensitive to calculate the focal depth (Zhao Zhu, 1992). Although the multi-layered crustal model and the integrated location with multi-path P and S waves can improve the precision of determination of focal depth (Wang Zhouyuan, 1989) the change of slowness is so sensitive that its results tend to deviate from the truth. Its own accuracy is also relevant to the velocity structure in the region. (2) Applying the dynamics method to improve the accuracy of the focal depth which can be done by determining the moment tensor and time function of the source with the inversion method, the accuracy of focal depth can be improved by synthetic seismograms and the fitting of the observed seismograms (Robert, 1993; Beck and Christensen, 1991; Sleny, 1992). It seems to mean that results could be more reliable and more accurate, but the accuracy of the focal depth depends on the approximation between the used model and the actual medium when calculating Green’s function (Xu Lisheng and Chen Yuntai, 1997). Velasco et al. (1993) thought that the velocity model and the assumed source location had an influence on the depth of the centroid position, the source duration and the estimation of seismic moment so even by means of the dynamics methods such as waveform inversion etc., it is still difficult to accurately measure the focal depth. In fact, due to the different methods and data, the accuracy of the focal depth is related to the focal depth, shear velocity, dip angle and slip angle (Anderson et al., 2009). So different researchers obtained different focal depths (Xu Lisheng and Chen Yuntai, 1997). (3) Some scholars used deep phases (surface reflection phases Pp and sP) to improve the accuracy of determination of focal depth (Stroujkova, 2009). Thinking this would help to reduce calculation errors of the focal depth caused by the uncertainty of seismic velocity. It is difficult to identify deep seismic phase. Only 11% of seismic events use deep seismic phase to obtain focal depths in the International Data Centre (IDC) (Stroujkova, 2009). It can therefore be seen from this that the use of deep seismic phase is restricted by many objective conditions and there are also many problems in the range of 300km for the identification of the deep seismic phase. This method is suitable for places where there is seismic array. (4) Some scholars believe that the double difference location method can effectively eliminate common propagation path effects from source to station and have little influence by the velocity model for it uses the difference between travel-time of signals to inverse the source location. The focal depth is therefore more reliable (Waldhauser & Ellsworth, 2000). However, due to the poor similarity among the signals, the low signal-to-noise ratio and the value range of trigonometric functions amongst other reasons, the calculated time difference is sometimes not accurate when using the cross spectrum method in calculating the signals (Liu Jinsong et al., 2007). This method is generally applicable to the arrays and precise location of swarms or aftershock sequences. (5) Another method is the G-R relation with statistics of the depth, namely
using statistical methods to improve the estimation of focal depth (Jessie et al. [2002]). This calibrates the magnitudes of the earthquakes which deviate from the GR curve according to the depth until the corrected magnitude meets the GR relationship, thus giving us a statistical focal depth. Such focal depth is significant to understand seismic tectonic settings and interpret seismic nucleation mechanisms. At present, this method is only used in scientific research and it is difficult to apply to daily earthquake rapid reporting and the handling of the earthquake catalog.

Although many methods can be used to determine focal depth, all of these methods involve traveling time wave velocity and crustal model. Therefore, studying the fine structure of a seismically active area is one of the prerequisites in improving the accuracy of the focal depth. The most important is that the epicenter has to be surrounded by a dense regional seismic network and there is at least one station with an epicentral distance that is less than the focal depth. This will greatly enhance the determination accuracy of focal depth (Stein et al. [1986]). Therefore, immediately setting up mobile seismic observation stations (network) in the source region after an strong earthquake is an effective method to amend the depth of the main shock.

The focal depth issue involves the understanding of the fault rupture process which has become a big concern. For example, the focal depth of the Wenchuan earthquake has aroused the concern of scholars. In China, the publication of earthquake catalogs and earthquake quick reports rely on observation data of regional seismic networks. In this paper, according to travel time formula of near earthquakes, we discussed the influences of epicentral distance arrival-time differences and velocity models on the focal depth.

1 THEORETICAL ESTIMATION OF FOCAL DEPTH ERROR

The travel time formula of near earthquake is:

\[ t_{pi} = \frac{\sqrt{\Delta_i^2 + h^2}}{V_p} \]  

(1)

Where \( i \) expresses different receiving points, \( t_{pi} \) expresses the travel time of P wave, \( V_p \) is velocity of P wave, \( \Delta_i \) expresses epicenter distance of the \( i \)th station, \( h \) expresses focal depth.

Focal depth derived from the formula (2) is:

\[ h^2 = \frac{i_{p}^2 V_p^2}{\Delta_i^2} \]

Differentiating both sides of the above formula gets the relationship between the depth \( h \) and its variation \( dh \), the travel time \( t_{pi} \) and its variation \( dt_{pi} \), the epicentral distance \( \Delta_i \) and its variation \( d\Delta_i \), P wave velocity \( V_p \) and its variation \( dV_p \) as:

\[ dh = \frac{i_{p}^2 V_p dV_p + V_p^2 t_{pi} dt_{pi} - \Delta_i d\Delta_i}{h} \]

(2)

\[ dh = i_{p}^2 V_p V_p^2 t_{pi} dt_{pi} - \Delta_i d\Delta_i \sqrt{i_{p}^2 V_p^2 - \Delta_i^2} \]

(3)

Formula (3) shows that the change of focal depth error is a composite function of travel time and travel time residuals velocity and velocity errors epicentral distance and the epicentral distance error. It reflects the complexity of determining focal depth error. According to formulae (2) and (3), we analyzed the change rules of \( dh \) and the impact of the focal depth on \( dh \) under the condition of \( dV_p = 0 \), \( d\Delta_i = 0 \), \( dt_{pi} = 0 \) and \( dV_p = 0 \), \( d\Delta_i = 0 \), \( dt_{pi} = 0 \).

(1) Supposing that the errors of epicentral distance \( \Delta_i \) and velocity model are zero, the observation travel time error has an impact on the focal depth, namely \( dV_p = 0 \), \( d\Delta_i = 0 \), \( dt_{pi} = 0 \) at this time the formula (2) becomes:
\[ dh = \frac{V_p^2 \Delta t_p \cdot dt_{pi}}{\sqrt{V_p^2 - \Delta_i^2}} \]  
\[ = \frac{V_p \sqrt{\Delta_i^2 + h^2}}{h} \cdot dt_{pi} \]  

Formula (5) reflects that the change of focal depth error \( dh \) is related to the epicenter distance, travel time residuals, and the depth.

If \( \Delta t_p = 0.0 \text{s} \) between the theoretical arrival time and the actual one, then \( dh = 0 \). When the arrival time error is \( \Delta t_p \) between the theoretical arrival time and the actual one, it is seen from formula (4) that the depth error is constrained by three factors, i.e. the epicenter distance \( \Delta_i \), the travel time (or path) and the arriving time error \( \Delta t_p \), and the velocity model error (or crustal model). Impacts of these factors on the focal depth are nonlinear.

To simplify the discussion, we take formula (5) as an example. Supposing the crust is uniform medium the depth \( h = 10 \text{km} \), \( V_p = 6.0 \text{km/s} \), theoretical epicentral distance \( \Delta_i = 1\text{5} \), \( 10\text{2} \), \( 20\text{5} \), \( 30\text{0} \), \( 40\text{0} \) km (equivalent to the stations distribution in different distances). Fig. 1 shows the different travel-time residual errors caused by the focal depth errors. It can be seen from the curve that the travel time residual error has great influence on depth error. If coupled with observation errors, the discussion of depth error will become more complicated. When the travel time residual is 0.05 s at the station at 100 km, depth error reaches \( \pm 3.66 \text{km} \); when the travel time residual is 0.1 s, depth error reaches \( \pm 6 \text{km} \). Actual residual error of travel time tends to be greater than 0.1 s therefore the credibility of depth is very low and can be used only as reference. In addition, Fig. 1 shows that depth error will increase with the increased epicentral distance or station location. It also means that data from near stations must be used to calculate the depth.

![Fig. 1](image)

The depth error caused by the travel time residuals at different epicentral distances (\( V_p = 6.0 \text{km/s} \))

(2) Supposing that the epicentral distance \( \Delta_i \), travel time residual are zero error, inspecting how observation travel time error has an impact on the focal depth, namely \( dt_{pi} = 0 \), \( d\Delta_i = 0 \).
\( dV_p \neq 0 \) at this time, the formula (2) becomes:

\[
dh = \frac{t_i^2 V_p dV_p + V_p^2 t_p d\Delta \Delta}{h} = \frac{t_i^2 V_p^2 dV_p}{h} = \frac{\Delta_i^2 + h^2}{V_p h} dV_p
\]  

(6)

It can be seen from the formula (6) that the error of the velocity model also has a certain influence on the depth without considering epicentral error and under the condition of the observation error being zero. To simplify the discussion, supposing crust is uniform medium, the depth \( h = 15 \text{km} \), \( V_p = 6.0 \text{km/s} \), theoretical epicentral distance \( \Delta_i = 15 \text{km}, 10 \text{km}, 20 \text{km}, 30 \text{km}, \ldots, 100 \text{km} \).

Fig. 2 shows the results. It can be seen from Fig. 2 that the depth error will increase with the increasing of the epicentral distance and velocity.

\[\text{Fig. 2}\]

The depth error in different epicentral distances caused by the change of velocity (crustal model) (assuming the depth \( h = 15 \text{km} \))

(3) Assuming that the velocity model is accurate and the observed travel time is error-free, the horizontal location error has an influence on the depth error, namely, when \( dV_p = 0 \), \( dt_p = 0 \), \( d\Delta_i \neq 0 \), then the formula (2) becomes:

\[
dh = \frac{-\Delta_i}{h} d\Delta_i
\]

(7)

It can be seen from formula (7) that even if the velocity model is accurate and the observed travel time is error-free, the greater the horizontal error of the location is, the greater the depth error will be. The depth error is inversely proportional to the depth.

With formula (7), we can analyze qualitatively and quantitatively the influence of Class I, II, and III location precision (horizontal error) on the depth error. As known from location precision that horizontal error of Class I, II, and III location precision is perhaps 5 km, 10 km, and 20 km respectively. Under the condition of such horizontal errors, the depth error will have relatively large change with the distances of stations. In this paper, only the change of depth error under certain conditions is discussed (see Fig. 3).

(4) Under normal circumstances, \( dV_p, d\Delta_i, dt_p \) are not zero, namely, \( dt_p \neq 0 \), \( d\Delta_i \neq 0 \), \( dV_p \neq 0 \), then the formula (2) can be written as:
The influence of horizontal location error on the depth error

It can be seen from Fig. 3 that the greater the horizontal error the greater the depth error.

\[
d h = \frac{t_{pi} V_p d V_p + V_p t_{pi} \Delta_i \Delta, d \Delta,}{h} = \left( \frac{\Delta^2_i + h^2}{V_p} \right) d V_p + V_p \sqrt{\Delta^2_i + h^2} dt_{pi} - V_p \Delta_i d \Delta, \tag{8}
\]

We discuss four kinds of cases: For the first three cases given the depth \( h = 15 \text{km} \), \( V_p = 6 \text{km/s} \):

(1) When \( dt_{pi} = [0.01, 0.5, 1.0] \) s, \( d V_p = 0.5 \text{km/s} \), \( d \Delta, = 5 \text{km} \) the change of \( dh \) is shown as in Fig. 4.

It can be seen from Fig. 4 that under the condition that the errors of velocity model and horizontal location are given the travel time residual has a larger influence on the depth error.

(2) When \( dt_{pi} = 0.5 \) s, \( d V_p = [0.1, 0.5, 1.0] \) km/s, \( d \Delta, = 5 \text{km} \) the change of \( dh \) is shown as in Fig. 5.

It can be seen from Fig. 5 that under the condition that the travel time residual and horizontal location errors are given the error of the velocity model has a larger influence on the depth error.

(3) When \( dt_{pi} = 0.5 \) s, \( d V_p = 0.5 \text{km/s} \), \( d \Delta, = [5, 10, 20, 50] \) km the change of \( dh \) is shown as in Fig. 6.

It can be seen from Fig. 6 that under the condition that the travel time residual and the velocity errors are given the horizontal location error has a larger influence on the depth error.

(4) The influence of the depth on the depth error.

When \( dt_{pi} = 0.5 \) s, \( d V_p = 0.5 \text{km/s} \), \( d \Delta, = 5 \text{km} \), \( h = [5, 15, 25] \) km the change of \( dh \) is shown as in Fig. 7.

It can be seen from Fig. 7 that the shallower the earthquake the greater the depth error. This conclusion is consistent with many location results of deep earthquakes (Stein et al. 1986).

Fig. 3

The influence of horizontal location error on the depth error

\[
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Fig. 3 illustrates that the depth error will increase with the increase of the epicentral distance or station distance which means the data from near stations should be used as far as possible in determining the focal depth or location of an earthquake. Using data from long-distance stations
The influence of different travel time residuals on the depth error

![Fig. 4](image)

The influence of different travel time residuals on the depth error

should be avoided in determining earthquake locations, which will affect the accuracy of calculating depth. We should be cautious about it in actual work. As shown in Fig. 7 under the same conditions, the depth error of $h = 5\text{km}$ will be much greater than the one of $h = 15\text{km}$ with an increased station distance.

![Fig. 5](image)

The influence of the change of $V_p$ velocity on the depth error
2 DISCUSSION

It can be seen from formula (4) that the depth itself is related to epicentral distance $\Delta$, the travel time (or propagation paths) and velocity model (crustal model) and yet the travel time is related to the origin time and observation accuracy. In the above discussion we all assume that the
observation error is zero, but in actual fact, whether by automatic identification or manual analysis, the observation error of the arrival times of the primary wave and subsequent seismic phases identified by different analysts are possible. They have an influence on calculating the depth.

Earthquake location results are obtained under the conditions of a given model. A certain amount of error exists between the model itself used in location and the real crust model. Therefore, the error of the original time will exist, which will cause travel time error, which finally leads to depth error.

These results are discussed only according to P wave travel time equations of near earthquakes and assuming that the crust is uniform. The real crust is with a layered characteristic and the crustal media are inhomogeneous and anisotropic.

In the multi-layered model, the crustal thickness of each layer is uncertain, the observed primary phase perhaps is Pn wave. At this moment, depth error is not only constrained by the three factors of epicentral distance, the travel time (or propagation path) and velocity model (crust model), but also the error of crustal thickness, where the source is in which the latter will have an influence on depth error. In addition, the non-horizontal variation of crustal thickness makes the discussion more complicated. This is also one of the reasons why the depth is one of the parameters which is very difficult to calculate exactly in seismology.

Credibility of focal depth is discounted when no station with an epicenter distance less than the focal depth receives the seismic waves.

3 CONCLUSION

Through the above discussion, we draw the following conclusions:

1. The precision error of focal depth of near earthquakes is constrained by epicentral distance Δ, horizontal location error, travel time and the travel time error Δt_p, the difference of velocity model (crustal model), and the size of focal depth and so on. These factors have a nonlinear influence on the focal depth. When the propagation speed of seismic wave is constant, the depth error increases with the increased epicentral distance and travel time residuals. The shallower the earthquake is, the greater the depth error will be.

2. When the travel time residuals are constant, the focal depth error increases as the epicenter distance and seismic wave velocity increase.

3. When the speed is known and the travel time residuals are constant, the shallower the earthquakes, the greater the location error. This conclusion is consistent with many location results of deep earthquakes (Stein el al. 1986).

4. The horizontal error associated with Class I, II and III location precisions increases with the increase of epicentral distance Δ, travel time residual Δt_p, and velocity model error, and depth error will increase too.

4 INSPIRATION

Although the seismic source depth is one of the parameters which are very difficult to be accurately determined in seismology, and the results obtained by various methods are different, comparative analysis can be done on these results, which may improve the accuracy of focal depth determination. Setting up mobile observation seismic stations (network) in the source region immediately after an earthquake is an effective method of amending the accuracy of the mainshock depth.
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Zhang Chaojun born in 1965 is an associate research professor of the China Earthquake Networks Center. He obtained his Doctor’s degree in the Graduate University of the Chinese Academy of Sciences in 2008. His major is seismology. E-mail: zhangchaojun@seis.ac.cn; zcj72@hotmail.com