The Conversion Relationship between the Local Earthquake Magnitude and Surface Wave Magnitude in the Inner Mongolia Digital Seismic Network

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Using 116 earthquakes over $M_L$ 3.8 in the Inner Mongolia region from 2008 to 2015, the local earthquake magnitude $M_L$ and surface wave magnitude $M_S$ are remeasured. Based on norm linear regression (SRT and SRT2) and norm (OR) orthogonal regression method, we established the conversion relationship between $M_L$ and $M_S$. The results were tested with Gaussian disturbance. The result shows that the orthogonal regression method (OR) result has the best fitting curve, and the conversion relation is $M_S = 0.96 M_L - 0.10$. The difference between our result and Guo Lücan’s ($M_S = 1.13 M_L - 1.08$) may be caused by regional tectonic characteristics. $M_S$ Inner Mongolia value is significantly higher than the $M_S$ empirical value, with an average difference of 0.23, the difference distribution of empirical relation and the rectified relation is in the range of 0.2–0.3.

Key words: Local earthquake magnitude $M_L$; Surface wave magnitude $M_S$; Gauss perturbation; Magnitude conversion

INTRODUCTION

Seismic monitoring is the most important observation basis for earthquake prediction research. In which, earthquake magnitude is an important parameter in earthquake prediction and other

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related earthquake studies (Chen Yuntai et al., 2000, 2004; Liu Ruifeng, 2003). Both current earthquake prediction methods and empirical precursory formulas are directly related to earthquake magnitude. Two scales of earthquake magnitude are often used in earthquake research, namely, local earthquake magnitude $M_L$ and surface wave magnitude $M_S$ (Zhang Hongzhi et al., 2007; Wang Suyun et al., 2010). Since the 1970s, the empirical formula $^2(M_{S_{\text{empirical}}} = 1.13 M_L - 1.08$ (Epicentral distance $\Delta \leq 1000$ km)) obtained by Guo Lücan in 1971 which is applicable to North China has been widely adopted in China. Because different scales of earthquake magnitudes express different meanings, there is no conversion between different magnitudes in actual earthquake monitoring. However, in practice, empirical formulas for the conversion between different scales of magnitude are often put forward according to different needs. For example, earthquake magnitude $M$ is given in large-earthquake prompt reports of regional seismic networks and earthquake catalogue, and earthquake magnitude $M$ used in earthquake prediction and related earth science research are all converted from the empirical formulas. However, based on many years of practical work, the measured results of local earthquake magnitude $M_L$ and surface wave magnitude $M_S$ show that the conversion relation between these two scales does not conform to the current empirical formula, and there is systematic deviation which needs to be corrected.

The methods of norm linear regression and norm orthogonal regression are most commonly used to obtain a good conversion relationship between local earthquake magnitude $M_L$ and surface wave magnitude $M_S$. Theoretically, the study of the relation between different magnitude scales using the orthogonal regression method should be closer to the magnitude actually measured (Madansky A., 1959; Fuller W.A., 1987; Carroll R.J. et al., 1996; Liu Ruifeng et al., 2007). Many Chinese researchers used linear regression and orthogonal regression methods to explore the conversion relationship between local earthquake magnitude $M_L$ and surface wave magnitude $M_S$ in different regions, and obtained reasonable results. Liu Ruifeng et al. (2006, 2007) conducted a comparative study on earthquake magnitudes measured by the China Earthquake Networks Center with Earthquake Networks Center of the United States and China Earthquake Networks Center. Ren Kexin et al. (2008) made a comparative analysis of the new magnitude scale of IASPEI and the traditional magnitude scale. Xie Zhuojuan et al. (2012) studied the empirical relationship between surface wave magnitude and local earthquake magnitude in the Chinese mainland and adjacent regions. Liu Guohua et al. (2006) and Yang Jingqiong et al. (2013) studied the conversion relationship between local magnitude and surface wave magnitude in the Yunnan region, and Zhang Cheng (1981) explored the conversion relationship between local magnitude and surface wave magnitude in Gansu.

The Inner Mongolia Autonomous Region spans the northeast, north and northwest regions of China, and is located at the northern extreme of the North-south seismic belt and other important tectonic regions, where the geological structure is complex and seismic activity is intense (Cao Gang, 2001). Many years of local magnitude measuring work shows that systematic deviation between these two kinds of magnitude scales brings great confusion to seismological researchers, with the enactment of the new General Ruler for Earthquake Magnitude. In accordance with the principle of “inheritance” of magnitude measurement, in order to achieve a “seamless” link-up between the old and new magnitude scales, on the basis of the new magnitude scale and methods of norm linear regression and norm orthogonal regression, using digital waveform data of 116 events with $M_L \geq 3.8$ is also measured using digital 2008 – November 2015 based on the measurement of local earthquake magnitude $M_L$ and surface wave magnitude $M_S$ to obtain the

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conversion relationship between local magnitude and surface wave magnitude, which is compared with the empirical relationship obtained by Guo Lücan, who hopes to provide fundamental data for earthquake prediction and seismological research in Inner Mongolia.

1 DATA SELECTION

Since the completion of the digital observation network of the tenth “Five-year Plan” of Inner Mongolia, the digital monitoring network composed of 81 digital stations (39 in Inner Mongolia and 42 in its neighboring provinces) has covered most parts of Inner Mongolia and the monitoring level has improved significantly, which provides a necessary data basis for correcting the conversion relationship between local magnitude and surface wave magnitude and establishing the new conversion relationship between local magnitude and broadband surface wave magnitude. In this study, digital waveform data of 116 $M_L \geq 3.2$ earthquakes are selected to establish the conversion relationship between local magnitude and surface wave magnitude (Fig. 1), and waveform data of 46 $M_L \geq 4.5$ earthquakes are used to establish a conversion relationship between local magnitude and broadband surface wave magnitude (Fig. 2).

2 MEASUREMENT OF EARTHQUAKE MAGNITUDE

Local earthquake magnitude $M_L$, surface wave magnitude $M_S$ are all determined in accordance with the new General Ruler for Earthquake Magnitude (GB17740–2017) (Liu Ruifeng et al., 2017).
Fig. 2  Distribution map of epicenters of the 46 earthquakes with $M_L \geq 4.5$

2.1 Local Earthquake Magnitude $M_L$

Local magnitude $M_L$ is calculated by the following formula (Liu Ruifeng et al., 2015) using the maximum amplitude of S or Lg wave recorded by analog short-period seismograph DD-1.

$$ M_L = \lg A + R(\Delta) $$

(1)

where, $A$ is the maximum horizontal ground motion displacement, $A = (A_S + A_E)/2$, unit; $m$, $A_S$, and $A_E$ are the maximum amplitudes of S or Lg wave in SN and EW direction respectively, Unit; $m$, and $R(\Delta)$ is the empirical formula for local earthquake magnitude.

2.2 Surface Wave Magnitude $M_S$

With respect to surface wave magnitude $M_S$ for shallow-focus earthquakes ($h \leq 60\,\text{km}$), the original broadband records are simulated as records of SK medium-long period seismograph and the maximum amplitude of surface wave ground motion displacement and corresponding period are used for calculation with the following formula (Liu Ruifeng et al., 2015).

$$ M_S = \lg \left( \frac{A}{T} \right) + 1.66 \lg(\Delta) + 3.5 $$

(2)

where, $A$ is the maximum amplitude of ground motion displacement of seismic surface wave, which is the vector sum of ground motion displacements in two horizontal directions, unit; $m$, $\Delta$ stands for epicentral distance, within a scope of $2^\circ < \Delta < 130^\circ$, and $T$ is the corresponding amplitude for $A$, unit; $s$. Calculation formula is as below (Liu Ruifeng et al., 2015).

$$ T = \frac{r_{A_S} + r_{A_E}}{A_S + A_E} $$

(3)
3 REGRESSION METHOD

Based on methods of 1 norm linear regression and 2 norm orthogonal regression (Carroll R.J. et al., 1996), the conversion relationships between $M_L$ and $M_s$, $M_L$ measured by Inner Mongolia Digital Seismic Network are analyzed.

3.1 The Linear Regression Method

For two or more random variables with statistical correlation, quantitative statistical relations between them can be determined based on a large number of observation data, that is, certain mathematical formula can be obtained to express these relations, which is called the regression equation (Liu Ruifeng, 2007). $N$ data points $(x_i, y_i)$, $(i = 1, 2, 3, \cdots, N)$ are intended to be fitted into a linear model as shown below

$$Y = AX + B \quad (4)$$

As it is commonly known, for the same sample point, different regression equations can be obtained with the selection of different independent and dependent variables, that is, fitting in $X$ or $Y$ direction will result in different regression lines (Sun Yanqing, 2002; Liu Ruifeng, 2007). To determine $A$ and $B$ in linear empirical formula $Y = AX + B$, we should first judge the error direction according to measurement error, and then use the error direction to determine the fitting direction (Huang Jie et al., 2000; Sun Yanqing, 2002).

In general, the linear least square regression method (SR) is used to determine the coefficients $A$ and $B$. Gutenberg et al. (Gutenberg B., 1945a, 1945b) provided the relation between $M_s$ and $M_L$ using the SR method. SR regression method is suitable for the case that the deviation of one variable is larger than that of another variable. When determining coefficients $A$ and $B$ to fit formula (4), there are two possibilities as follows (Draper N.R. et al., 1998).

The first possibility:

$$SR_1 \quad Y \leftarrow A_1X + B_1 \quad (5)$$

Application condition is $\sigma^2_{xx} \rightarrow 0$ and $\sigma^2_{xy} > 0$, where $\sigma^2_{xx}$ and $\sigma^2_{xy}$ are respectively variances of $x$ and $y$.

The second possibility:

$$SR_2 \quad Y \rightarrow A_2X + B_2 \quad (6)$$

This case is also known as de-normalization regression (Ruppert et al., 1996) and the application condition is $\sigma^2_{xx} > 0$ and $\sigma^2_{xy} \rightarrow 0$.

3.2 The Orthogonal Regression Method

When both variables are likely to change greatly, that is, when measurement errors in $X$ and $Y$ directions are both not negligible, fitting can’t be carried out in a single direction. In this case, the fitting line should satisfy the condition that the sum of squares of vertical distances between each measuring point and the fitting line should be the minimum, that is, orthogonal fitting (Wu Junlin et al., 1992; Liu Wei et al., 1994; Li Xiongjun, 2005a, 2005b; Jiang Hui et al., 2006). The orthogonal regression method can overcome the disadvantage of poor fitting stability caused by fixed single direction optimization. OR is used to represent the orthogonal regression method, then

$$OR \quad Y = A_3X + B_3 \quad (7)$$

The Hesse representation is usually used to represent orthogonal regression (Ruppert et al., 1996), that is, both variables are put on the right side of the equal sign, which means that both
variables on the right size are changing, namely

\[ P = n_1 X + n_2 Y \]  \hspace{1cm} (8)

where, \( P = B_3/q \), \( n_1 = A_3/q \), \( n_2 = 1/q \), \( q = (1 + A_3^2)^{1/2} \) and \( -n_1/n_2 = A_3 \). The coefficients \( A_3 \) and \( B_3 \) of \( X \) and \( Y \) are divided by \( q \) to make them conform to the Hesse representation.

If equations (5) and (6) are applied to magnitude scales \( M_L \) and \( M_S \), the SR_1 method is applicable to the impact on \( M_s \) when the measurement deviation of \( M_s \) is large, while the SR_2 method is applicable to the impact on \( M_s \) when measurement deviation of \( M_s \) is large. However, since all magnitude measurements may have certain errors, whether it is SR_1 or SR_2, the results obtained are different from the actual magnitude. Therefore, theoretically, the study of the relations between various magnitude scales using orthogonal regression should be more close to the actual magnitude (Madansky A., 1959; Fuller W. A., 1987; Liu Ruifeng et al., 2007). For comparison, the relationship between different magnitude scales obtained by using 1 norm SR_1, SR_2 and 2 norm OR methods are given respectively in this paper, however, in the analysis and conclusion of the results, the fitting relationship is mainly based on the OR method.

4 CONVERSION RELATIONSHIP BETWEEN LOCAL MAGNITUDE AND SURFACE WAVE MAGNITUDE

Based on linear regression and orthogonal regression methods, the conversion relationship between local earthquake magnitude \( M_L \) and surface wave magnitude \( M_S \) is obtained. Meanwhile, Gaussian disturbance is used to perturb each point randomly, and data after perturbation is analyzed by regression. The final results are compared with the empirical relationship obtained by Guo Lücan (1971).

4.1 Conversion Relationship between \( M_L \) and \( M_S \)

4.1.1 Linear Regression and Orthogonal Regression

Based on the general linear regression (SR_1 and SR_2) and orthogonal regression methods (OR), linear regression is conducted for the re-measured magnitude \( M_L \) and \( M_S \) of 116 earthquakes (altogether 329 magnitude pairs) during 2008–October, 2015, and the magnitude range used for linear regression is \( 3.8 \leq M_S \leq 6.7 \). Table 2 shows the relation between \( M_L \) and \( M_S \) obtained by linear regression and orthogonal regression, and Fig.3(a) provides 4 relation curves between \( M_L \) and \( M_S \) obtained by linear regression, orthogonal regression (OR) and Guo Lücan’s empirical equation \( M_S \text{empirical}=1.13M_L-1.08 \). As can be seen from Fig.3(a), the curve obtained by the orthogonal regression method (OR) is located in the middle of data points, and the root-mean-square error is the minimum, which is the optimal fitting curve. Thus, the conversion relationship between the corrected local magnitude \( M_L \) and surface wave magnitude \( M_S \) in Inner Mongolia is as below:

\[ M_S \text{Inner Mongolia} = 0.96M_L - 0.10 \]  \hspace{1cm} (9)

Fig.3(b) shows the quantitative distribution of the difference between \( M_S \text{Inner Mongolia} \) and \( M_S \text{empirical} \* \). As can be seen from Fig.3, most of the difference values are distributed between 0.0 and 0.4, among which, \( M_S \text{Inner Mongolia} - M_S \text{empirical} \* = 0.3 \) is the majority. It can be seen that the measured \( M_S \text{Inner Mongolia} \) of Inner Mongolia earthquakes differs greatly from empirical \( M_S \text{empirical} \* \).
Table 2  Relationship between $M_L$ and $M_s$

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>The number of earthquakes</th>
<th>Range of magnitude</th>
<th>Regression method</th>
<th>Relation</th>
<th>$RMS$ (root-mean-square error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_L - M_s$</td>
<td>116</td>
<td>$3.8 \leq M_s \leq 6.7$</td>
<td>SR1</td>
<td>$M_s = 0.80 M_L + 0.60$</td>
<td>$\pm 0.32$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SR2</td>
<td>$M_s \rightarrow 1.16 M_L - 1.02$</td>
<td>$\pm 0.34$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OR</td>
<td>$M_s = 0.96 M_L - 0.10$</td>
<td>$\pm 0.24$</td>
</tr>
</tbody>
</table>

Fig. 3  Magnitude relationship and distribution of magnitude difference value

(a) Relationship between $M_L$ and $M_s$;
(b) Quantitative distribution of difference between $M_s$ Inner Mongolia and $M_s$ empirical

Fig. 4  The relationship between local magnitude $M_L$ and surface wave magnitude $M_s$ after Gaussian disturbance
4.1.2 Gaussian Random Disturbance

Using the built-in function Normand of the Matlab platform, the above 329 $M_l$ and $M_s$ magnitude pairs are adopted to conduct Gaussian disturbance on 329 surface wave magnitude $M_s$ points of 116 earthquakes. In the disturbance process, the mean value of magnitude $M_s$ added is 0 and the variance is 0.2. Meanwhile, linear regression (SR1 and SR2) and orthogonal regression (OR) are conducted for the data after the Gaussian disturbance. Table 3 shows the relationships between $M_l$ and $M_s$ obtained by linear regression and orthogonal regression, and Fig.4 provides four curves obtained by linear regression (SR1 and SR2), orthogonal regression (OR) and Guo Lücan’s empirical equation.

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>The number of earthquakes</th>
<th>Range of magnitude</th>
<th>Regression method</th>
<th>Relation</th>
<th>RMS( Root-mean-square error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_l-M_s$</td>
<td>116</td>
<td>$3.8 \leq M_s \leq 6.7$</td>
<td>SR1</td>
<td>$M_s=0.79 M_l+0.66$</td>
<td>$\pm 0.37$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SR2</td>
<td>$M_s=1.23 M_l-1.31$</td>
<td>$\pm 0.37$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OR</td>
<td>$M_s=0.98 M_l-0.16$</td>
<td>$\pm 0.27$</td>
</tr>
</tbody>
</table>

As can be seen from Table 3 and Fig. 4, the conversion relationship between local magnitude $M_l$ and surface wave magnitude $M_s$ has little change before and after a Gaussian disturbance. The coefficient $A$ increases by 0.02 from 0.96 to 0.98 and the coefficient $B$ increases by 0.06 from 0.10 to 0.16.

4.2 Comparative Analysis of the Conversion Relationship Between Two Magnitude Scales

According to the two kinds of magnitude conversion relationships of formula (9) and Guo Lücan’s empirical formula $M_s^{\text{empirical}} = 1.13 M_l - 1.08$, measured value of two kinds of conversion relationships $M_s^{\text{empirical}}, M_s^{\text{Inner Mongolia}}$ and distribution of the difference value between $M_s^{\text{empirical}}$ and $M_s^{\text{Inner Mongolia}}$, it can be seen that:

1. As a whole, there is a great difference between the two kinds of magnitude conversion relationships. The coefficient $A$ does not change much, decreasing by 0.07 from 1.13 to 0.96, while coefficient $B$ varies greatly and there is obvious systematic deviation between the two. The difference may be caused by the difference of regional structural characteristics.

2. The value of $M_s^{\text{Inner Mongolia}}$ converted from $M_l$ measured in this study by formula (9) is significantly higher than the value of $M_s^{\text{empirical}}$ converted by $M_s^{\text{empirical}} = 1.13 M_l - 1.08$, with an average difference of 0.23. Therefore, formula (9) is recommended.

3. Fig.3 (b) shows the quantitative distribution of the difference between $M_s^{\text{empirical}}$ and $M_s^{\text{Inner Mongolia}}$. It can be seen from Fig. 3(b) that the magnitude difference of most earthquakes is 0 $\sim$ 0.4, and the number of earthquakes with difference within 0.2 $\sim$ 0.3 is the largest.

4. Table 4 shows the comparison between relation $M_s^{\text{empirical}}$ and $M_s^{\text{Inner Mongolia}}$ within magnitude range of $3.0 \leq M_l \leq 7.0$. It can be seen from Table 4 that the magnitude difference between them ranges from 0 to 0.5.
Table 4  The comparison between relation $M_{S \text{ empirical}}$ and $M_{S \text{ Inner Mongolia}}$ within magnitude range of $3.0 \leq M_L \leq 7.0$

<table>
<thead>
<tr>
<th>$M_L$</th>
<th>$M_{S \text{ Inner Mongolia}}$</th>
<th>$M_{S \text{ empirical}}$</th>
<th>$M_{S \text{ Inner Mongolia}} - M_{S \text{ empirical}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>2.8</td>
<td>2.3</td>
<td>0.5</td>
</tr>
<tr>
<td>3.5</td>
<td>3.3</td>
<td>2.9</td>
<td>0.4</td>
</tr>
<tr>
<td>4.0</td>
<td>3.7</td>
<td>3.4</td>
<td>0.3</td>
</tr>
<tr>
<td>4.5</td>
<td>4.2</td>
<td>4.0</td>
<td>0.2</td>
</tr>
<tr>
<td>5.0</td>
<td>4.7</td>
<td>4.6</td>
<td>0.1</td>
</tr>
<tr>
<td>5.5</td>
<td>5.1</td>
<td>5.1</td>
<td>0.0</td>
</tr>
<tr>
<td>6.0</td>
<td>5.6</td>
<td>5.7</td>
<td>0.1</td>
</tr>
<tr>
<td>6.5</td>
<td>6.1</td>
<td>6.3</td>
<td>0.2</td>
</tr>
<tr>
<td>7.0</td>
<td>6.5</td>
<td>6.8</td>
<td>0.3</td>
</tr>
</tbody>
</table>

5 CONCLUSIONS

Based on norm linear regression ($SR_1$ and $SR_3$) and norm orthogonal regression (OR) methods, remeasuring local magnitude $M_L$ and surface wave magnitude $M_S$ of 116 $M_L \geq 3.8$ earthquakes in Inner Mongolia during 2008—November 2015, Guo Lücan’s empirical magnitude conversion relation is corrected and the conversion relationship between local magnitude $M_L$ and surface wave magnitude $M_S$ in Inner Mongolia is obtained. We can draw the following conclusions.

(1) It can be seen from the relation curves between $M_L$ and $M_S$ obtained by linear regression ($SR_1$ and $SR_3$) and orthogonal regression (OR) methods that the curve obtained by the orthogonal regression method (OR) is located in the middle of data points, whose root-mean-square error is the minimum, which is the optimal fitting curve. Thus, the conversion relationship between local magnitude $M_L$ and surface wave magnitude $M_S$ in Inner Mongolia is $M_{S \text{ Inner Mongolia}} = 0.96M_L - 0.10$.

(2) Based on linear regression ($SR_1$ and $SR_3$) and orthogonal regression (OR) methods, Gaussian random disturbance is conducted for each point, and the conversion relationship between local magnitude and surface wave magnitude does not change much.

(3) By analyzing Guo Lücan’s magnitude conversion relation $M_S = 1.13M_L - 1.08$ and magnitude conversion relation $M_{S \text{ Inner Mongolia}} = 0.96M_L - 0.10$, it is known that there is obvious systematic deviation between the two, which may be caused by the difference of regional structural characteristics.

(4) The value of $M_{S \text{ Inner Mongolia}}$ is significantly higher than that of $M_{S \text{ empirical}}$, with an average difference of 0.2. The results show that Guo Lücan’s empirical conversion relation is no longer suitable for regional characteristics of Inner Mongolia, and it is suggested to use the newly corrected conversion relation between local magnitude $M_L$ and surface wave magnitude $M_S$.

(5) Quantitative distribution of difference value between $M_{S \text{ Inner Mongolia}}$ and $M_{S \text{ empirical}}$ shows that the magnitude difference of most earthquakes is 0–0.4, and the number of earthquakes with difference within 0.2–0.3 is the largest.

(6) It can be seen from the comparison table between relation $M_{S \text{ Inner Mongolia}}$ and $M_{S \text{ empirical}}$ within magnitude range of $3.0 \leq M_L \leq 7.0$, the magnitude difference between ranges from 0 to 0.5.

(7) There are not many $M_L \geq 3.8$ earthquakes in Inner Mongolia during 2008—November.
2015, which are unevenly distributed, therefore, the results of this study still needs to be improved in future practical use to make it more consistent with the regional characteristics of Inner Mongolia.

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