Mechanical Response of Saturated Geological Rock Mass under Tidal Force

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In this paper, the mechanical response of saturated geological rock under tidal force is explored by poroelastic theory. First, we use the free energy formula of saturated rock under a tidal force to study the relationships of pore pressure with stress and stress with strain. Then we analyze the relationship between rock strain and tidal potential by the equilibrium differential equations of saturated rock under tidal force. Finally, we derive the physical relationship between the two parameters (pore pressure and tidal mean stress) of saturated rock and tidal potential. The relationship shows that: pore pressure is directly proportional with tidal potential but tidal mean stress of saturated rock is inversely proportional with tidal potential. The ratio coefficient is related not only to the Lame coefficients of rock skeletons but also to the Biot modulus. By using this model to analyze observational well water level of C-18 well which locates in Huili, Sichuan Province, the well level response coefficient \((D)\) was estimated. This way, we derive the Skempton coefficient \((B)\), the coefficient \(A\) and \(C\) which refer to the response coefficients of pore pressure and tidal stress to tidal potential respectively. Then we compare the differences among each coefficient in coupling and uncoupling conditions. It shows that for saturated rocks, the response of stress and pore pressure to earth tides is a product of coupling and it is necessary to take into account the coupling effect when we study the mechanical response. The model will provide the basis not only for the study of mechanics and hydrodynamics of well-confined aquifer systems and the mechanics of faulting under tidal force but also for quantitative research of the triggering mechanism of tidal forces.

Key words: Poroelastic theory; Saturated geological rock mass; Pore pressure; Tidal stress; Tidal potential

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INTRODUCTION

Ever since the discovery of earth tides and related changes in water level, stress, strain, as well as pore pressure (water level) fluctuations generated by tidal forces on geological rock masses have been drawing researchers’ attention. Bredehoeft (1967) analyzed the relation between borehole water levels and earth tide strain. Based on poroelastic theory and groundwater dynamic principles, Zhang Zhaodong et al. (1988) deduced the response equation of well water level to earth tides. Wang Chengmin et al. (1988) discussed and concluded the expression of tidal stress on the sphere’s surface and investigated the changes in well water levels via equilibrium equations of deep well water level and tidal stress. Moreover, by means of the relationship between well water levels and tidal stress of oil or water bearing formation, Liu Yuansheng et al. (2000) inverted stress changes in this formation by observing water level changes in oil wells. However, certain geological rock masses like rupture zones in tectonic faults and confined aquifers have frequently been saturated by fluid (mainly water and various gases). As a result, under inner and outer pressure, this sort of rock interrelates and interacts with the fluid in the following way. It deforms to alter the pore volume so as to influence pore pressure; meanwhile, pore fluid affects rock deformation by means of pore pressure thus with the participation of fluid, the mechanical problem of geological rocks in the effect of tidal force, atmospheric pressure and tectonic stress is that of a fluid-solid coupling problem (Neuzil 2003).

Nevertheless, the disadvantage of the above research lies both in the unilateral recognition of influence of strain on pore pressure and in the ignorance of fluid effects when calculating stress and strain that is, it neglects the coupling effect of fluid with rock skeletons. Therefore, the introduction of fluid-solid coupling theory into problem analysis is highly necessary when studying earthquake-related fault mechanics in the effect of tidal forces as well as mechanics of a well-confined aquifer systems. On the foundation of fluid-solid coupling mechanisms, this article aims at investigating the physical relationships among pore pressure in rock, tidal stress and tidal potential and that between tidal stress (mean stress related to bulk strain) and pore pressure by means of research of the rocks’ mechanical responses to tidal forces. The model building lays foundation for both quantitative analysis on mechanics and hydrodynamics of a well-confined aquifer system as well as on earthquake-related fault mechanics and trigger mechanisms affected by tidal force.

1 FREE ENERGY, STRESS AND PORE PRESSURE EXPRESSION OF SATURATED ROCKS AFFECTED BY TIDAL FORCE

1.1 Theoretical Foundation

Under tidal forces, additional stress is imposed on geological rocks which deforms accordingly. Hydraulic pressure in the pores (or pore pressure) will change as a result which simultaneously counter-influence rock deformation. This physical phenomenon can be explained by adopting the fluid–solid coupling principle in isotropic elasticity theory put forward by Biot in 1941 and 1956. Poroelastic theory is founded upon generalized Hooke law by which free energy expression of porous medium per unit volume in isothermal conditions can be described as follows (Biot 1941, 1956; Hameil et al. 2005) regardless of inelastic deformations of continuous saturated media.

\[ p = \rho_1 gh \]  

(1)

In this expression, \( F \) refers to free energy, \( \lambda \) and \( \mu \) stand for Lame parameters of solid skeletons and \( M \) is the Biot modulus of saturated rock and is also the elastic parameter related to compressibility in the fluid–solid coupling process. \( \beta \) the Biot coefficient is also termed as the

**Table 1** Skempton coefficient experimental values of different rock types (Yan Rui et al., 2008)

<table>
<thead>
<tr>
<th>Rock type</th>
<th>Ruhr sandstone</th>
<th>Berea sandstone</th>
<th>Weber sandstone</th>
<th>Westerly granite</th>
<th>Charcoal granite</th>
<th>Tennessee marble</th>
<th>Compacted sand</th>
<th>Loose sand</th>
<th>Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0.88</td>
<td>0.62</td>
<td>0.73</td>
<td>0.85</td>
<td>0.55</td>
<td>0.51</td>
<td>0.99</td>
<td>0.998</td>
<td>1</td>
</tr>
</tbody>
</table>

The period of pore pressure fluctuation caused by high-frequency tidal force tends to be...
shorter in comparison with the time scales for “drainage” or hydraulic response to occur (Neuzil 2003) thus the fluid convection under tidal forces is omitted and rock mass can be treated as approximately undrained. That is $\zeta = 0$ is strictly satisfied when the sealing of geological rock mass is relatively good. Therefore equation (6) is simplified as:

$$P = - B\sigma_m = - BK_1 I_1$$  \hspace{1cm} (7)

According to equation (7) it is deduced that when mean stress increases and rock mass expands pore pressure decreases and it is in inverse proportion with mean stress. The proportional coefficient that is the Skempton coefficient is not only in relation with elastic parameter $\lambda$ and $\mu$ of the rock skeleton but also with the Biot modulus $M$. As far as saturated crust rock mass is concerned inelastic deformation under tectonic stress can be omitted at short time scales of tidal force loading and unloading. It is approximately in the state of elastic cycles $B$ becomes a variable along with the changes in tectonic stress. This is the timeliness of elasticity hypothesis of rock mass by which the Skempton coefficient can be applied to show the changes in rock quality and be treated as an early-warning parameter to judge the abnormality of tectonic stress.

1.2 The Responses of Tidal Mean Stress to Tidal Force

Take one unit volume of saturated rock as representative elementary volume (called REV for short) for mechanical analysis. For any coordinate system (three axes being $x_1, x_2, x_3$) there are equilibrium differential equations under tidal force (Fang Jun 1984):

$$\sum_i \frac{\partial \sigma_{ij}}{\partial x_j} + \rho \frac{\partial \psi_{ij}}{\partial x_j} = 0 \quad (i=1,2,3)$$  \hspace{1cm} (8)

In the midst of which $\sigma_{ij}$ is the stress component in the direction of the $x_j$ axis of stress vector acting on the $x_i$ axial plane $\rho$ is the density of saturated rock and this article neglects the changes caused by tidal force. Equation (8) is the equilibrium differential equation of REV in the effect of an axial component (in the expression of acceleration) of tidal force. If rock mass is regarded as undrained then stress in expression (3) of the fluid–solid coupling effect should be considered being expressed in displacement and when substituted into equation (8) the equilibrium differential equation can be transformed as (take axis $x_1$, for instance)

$$(\lambda + \mu + \beta^2 M) \frac{\partial I_1}{\partial x} + \mu \nabla^2 u + \rho \frac{\partial \psi_{ij}}{\partial x} = 0$$  \hspace{1cm} (9)

In which $I_1 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ is bulk strain and $u$, $v$ and $w$ are the displacement in the direction of $x_1$, $x_2$, $x_3$. Similarly analogous equation exists in the direction of $x_2$, $x_3$. Equation (9) is a non–homogeneous differential equation concerning displacement $u$, $v$, $w$ in full consideration of the fluid–solid coupling effect while the equilibrium differential equation (in the direction of $x_1$) of porous medium or saturated rock regardless of the influence generated by pore pressure is

$$(\lambda + \mu) \frac{\partial I_1}{\partial x} + \mu \nabla^2 u + \rho \frac{\partial \psi_{ij}}{\partial x} = 0$$  \hspace{1cm} (10)

Solving equation (10) a new formula can be deduced (Fang Jun 1984; Liu Yuansheng et al. 2000)

$$I_1 = \frac{2\rho}{19\lambda + 14\mu} \psi_{ij}$$  \hspace{1cm} (11)

Comparing equations (9) and (10) they bear the same formation thus have similar
solutions. As long as we use $\lambda' = \lambda + \beta^2 M$ in equation (9) to take the place of $\lambda$ in equation (10) then substitute it into equation (10) we can acquire the bulk strain expression of saturated rock under tidal force in the undrained condition:

$$I_1 = \frac{2\rho}{19(\lambda + \beta^2 M) + 14\mu} \psi_n \tag{12}$$

Above is the bulk strain of geological rock under tidal force in consideration of the fluid–solid coupling effect. When equation (4) and (12) are combined the expression of mean stress can be:

$$\sigma_m = A \psi_n$$

$$A = \frac{2\rho K_u}{19(\lambda + \beta^2 M) + 14\mu} \tag{13}$$

Substituting equation (13) into (7) yields the expression of pore pressure:

$$P = -B\sigma_m = -C\psi_n$$

$$C = AB = \frac{2\rho \beta M}{19(\lambda + \beta^2 M) + 14\mu} \tag{14}$$

Equations (13) and (14) are expressions of tidal mean stress and pore pressure response respectively in consideration of the fluid–solid coupling effect of saturated geological rock under tidal forces. They demonstrate that tidal mean stress of saturated geological rock mass is in direct ratio with tidal potential while pore pressure is in inverse proportion with tidal potential. As far as given rock is concerned its density can be regarded as invariants. Proportion coefficients ($A$ and $C$) are only related to elastic parameters ($\lambda$, $\mu$, and $M$) of saturated rock mass thus similar to the Skempton coefficient. $A$ and $C$ are variables during the seismogenic process.

Above expressions of mean stress and pore pressure under tidal forces have fully considered the effect of pore pressure on rock strain and effective stress that is the fluid–solid coupling effect of rock mass. In contrast the conclusion of tidal mean stress and pore pressure response when coupling is considered differs greatly from the result without it. Due to the existence of fluid which shares part of the stress the effective stress on rock skeletons decrease as a result while the tidal stress of saturated rock mass is constant under identical tidal force. Or else it is regarded that the increased bulk modulus (from $K$ to $K_u$) which decides rock deformation makes the actual bulk strain of saturated rock mass smaller in comparison with that in uncoupling condition. This can be clearly seen in Table 2.

**Table 2** Differences of mean stress and pore pressure with or without considering fluid–solid coupling

<table>
<thead>
<tr>
<th>Coupling/Uncoupling</th>
<th>Bulk strain</th>
<th>Mean stress</th>
<th>Pore pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupling</td>
<td>$\frac{2\rho \psi}{19(\lambda + \beta^2 M) + 14\mu}$</td>
<td>$\frac{2\rho \psi K_u}{19(\lambda + \beta^2 M) + 14\mu}$</td>
<td>$-\frac{2\rho \psi \beta M}{19(\lambda + \beta^2 M) + 14\mu}$</td>
</tr>
<tr>
<td>Uncoupling</td>
<td>$\frac{2\rho \psi}{19\lambda + 14\mu}$</td>
<td>$\frac{2\rho \psi K_u}{19\lambda + 14\mu}$</td>
<td>$-\frac{2\rho \psi \beta M}{19\lambda + 14\mu}$</td>
</tr>
</tbody>
</table>

Note: Uncoupling here refers to the process that fluid effect is not considered when bulk strain and mean stress are solved via equilibrium differential equations. In essence this is to admit that the influence exerted by strain on pore strain and mean stress in uncoupling conditions is the same as that of porous rocks without water.
2 MODEL APPLICATION EXAMPLES AND ANALYSIS

For a well-confined aquifer system, the response of well water levels to tidal forces is a typical fluid-solid coupling phenomenon and the coupling mechanical response of an aquifer is fully reflected in the change of water level. \( P \) as defined is equivalent to the hydraulic head multiplied by \( \rho_l g \) where \( \rho_l \) is the density of water set at \( 10^3 \text{kg/m}^3 \) so the response of water levels to tidal force can be put as follows:

\[
h = - D \frac{\psi}{g}
\]

\[
D = \frac{2\gamma\beta M}{19(\lambda + \beta^2 M)} + 14\mu
\]

In which \( g \) is gravitational acceleration; \( \gamma \) is the relative density of saturated aquifer rocks; \( \psi/g \) is the theoretical height equilibrium sea level tidal; \( D \) is a dimensionless coefficient. In the process of earthquake preparations due to the rheological behavior of rocks \( D \) is a variable. It can also be considered as quantitative criteria of abnormal recognition the effect of which is similar as traditional tidal efficiency.

2.1 The Choice of Well-level Observation Station and Data

Digital observation data of groundwater levels provide foundation for the application of this model and testing the response of water levels (pore pressure) to tidal force. This article is based on the research of well C-18 located in Huili, Sichuan Province (102. 06°E, 26. 31°N) which lies in the intersection between the Mopanshan-Xigeda fault and Ninghui fault. The depth of the well is 523. 28m the aquifers whose hydraulic conductivity is 0. 1457m/d are medium thick dolomitic marble between 364. 66m and 371. 35m and has a marble crush zone between 443. 29m and 460. 54m. The observation data esp. static well level and atmospheric pressure are quite good. Also, the well level gives a clear response to earth tides and seismic waves.

To avoid the affect of rain loading and to highlight the response of water levels to tidal forces we have chosen data from well C-18 observed between December 10 ~ 14 2007. Analysis data of the well water level and calculated tidal potential are shown in Fig. 1.

![Fig. 1](image-url)

Analysis data of well water level and calculated tidal potential
2.2 Regression Analysis of Observational Data and the Solving and Analysis to Different Coefficients

With the help of the natural poroelastic mechanics experiment of tidal force affecting aquifers we calculate the responding coefficient of well water level $D$ and the responding coefficient $(A, C)$ of tidal stress pore-pressure to tidal force and aquifer Skempton coefficient $B$. Specific progress and results are as follows:

Under the effect of tidal force well water level fluctuation is of relatively high frequency. So here we use first order differenced data to get rid of the affect of lows such as tectonic stress and the change in trend and also to exploit bivariate regression about tidal potential and atmospheric pressure to get rid of the effects of tidal atmosphere pressure which have the same frequency as the tidal force. The regression equation is $\Delta h = -\alpha \Delta p - D \Delta \psi / g$. Using data from the observational water level $h$, barometric pressure $p$ and the theoretical values of tidal potential $\psi$ of its aquifer we make a bivariate linear regression analysis of $\Delta h, \Delta p$ and $\Delta \psi / g$ using statistical software (in which atmospheric pressure is converted to the height of water column). The result of the regression analysis is: $\Delta h = -0.336 \Delta p - 0.183 \Delta \psi / g$, multiple correlation coefficient $R = 0.991$, the relative coefficient between tide potential and water level $R = 0.985$.

Setting the relative density of the aquifer $\gamma = 3.32$ Lame coefficient respectively as $\lambda = 0.74 \times 10^{12}$ Pa and $\mu = 0.63 \times 10^{12}$ Pa (reference from Fang Jun 1984) and using the hypothesis $\beta = 1$ from Terzaghi. Combining the regression result we can work out that $D = 0.183$, $M = 1.324 \times 10^{12}$ Pa then we can derive $K = 1.16 \times 10^{12}$ Pa, $K_u = 2.484 \times 10^{12}$ Pa. Therefore $K_u / K = 2.14$ which shows that when taking the coupling effect into consideration the effective volume of the rock is twice that under uncoupling conditions and the strain reduces about two-fold which also means that the effect is larger for pore pressure to rock strain under undrained conditions. Finally we put $\lambda, \mu$ and the derived $M$ into equations (13) and (14) and get the relations of the aquifer’s mean stress and pore pressure with tide potential respectively. They are $\sigma_m = 343.4 \psi$, $P = -183 \psi$. Combining the dimension of tidal potential we can work out that tidal mean stress and pore pressure of saturated rock is about $10^3$ Pa (magnitude). Then we can derive the aquifer Skempton coefficient $B = 0.533$ or the relation between pore pressure and mean stress as $P = -0.533 \sigma_m$.

The above coefficient is based on the consideration of the coupling effect. In a similar way according to the expression of each physical formula in Table 2 we can derive the values without the consideration of the coupling effect. From Table 3 we can see the distinction of coefficients between considering fluid-solid coupling and uncoupling:

<table>
<thead>
<tr>
<th>Coupling/Uncoupling</th>
<th>$M$</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupling</td>
<td>1.324</td>
<td>343.4</td>
<td>0.533</td>
<td>183</td>
<td>0.183</td>
</tr>
<tr>
<td>Uncoupling</td>
<td>0.630</td>
<td>336.6</td>
<td>0.352</td>
<td>183</td>
<td>0.183</td>
</tr>
</tbody>
</table>

It is clear from Table 3 that regardless whether fluid-solid coupling effect is considered or not the $M$ derived from water level and tide potential is high while a further derivation of $A$’s value is lower which means that the difference in tidal mean stress derived from both models is not significant while the Skempton coefficient differs a lot. However the Skempton coefficient of aquifer rock (marble) derived from our model is much closer to its lab value $B = 0.51$ which also makes our model more reliable by proving that the fluid’s affect to solids (of the affect to skeleton strain from pore pressure) should be fully considered in discussing the mechanical...
response of saturated geological rock masses to tidal force. At the same time, values given by the uncoupling model do not go with the theoretical formula $AB = C$ or against $P = -Br_m$ which means that the discard of the coupling effect in calculating stress is inaccurate. It also gives further prove that response of tidal stress and pore pressure to tidal force results from the coupling effect on saturated rock mass.

### 3 CONCLUSION AND DISCUSSION

Starting from fluid–solid coupling effects, this article discusses the physical relations amongst pore pressure in rocks, tidal stress and tide potential with the effect of tidal force as well as the relation between tidal stress and pore pressure and establishes a mechanical model with the response of saturated geological rock masses to tidal force. By applying this model to analyze the tidal response of well water levels to earth tide, the Biot modulus of aquifer rocks is released with the data of both observational well water levels and theoretical tide potential. Based on this, we get the value of response parameters $A$, $B$ and $C$ and also the Skempton coefficient by comprising previous models in which the accuracy of our model is verified.

However, there are also shortcomings in our model. The first one is the limitation of the hypothesis on isotropy of rocks. In fact, with the effect of tectonic stress, geological rock mass always behaves anisotropically which will also affect the derivation of the mechanical response of rock mass and the Skempton coefficient. The second one is the limitation of the hypothesis that the aquifer does not drain in the case study. With the participation of the wellhole, it will convect with the aquifer (Zhang Zhaodong et al. 2002). Thus, the rationality of this hypothesis lies in the relation between the stress change cycle and time scale of convection which is related to the hydrogeological parameters (e.g. the hydraulic conductivity) of the aquifer. Considering the length of this article, we do not discuss this issue in depth (though the rationality of the hypothesis that Huili Well does not drain is verified by the calculated Skempton modulus). As for the response of pore pressure and aquifer stress of geologic rock mass under draining conditions, they are beyond our model the limitation to which requires further research.

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### REFERENCES


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